RELAXATION FUNCTION OF POLYCRYSTALLINE METALS IN CONDITIONS OF PULSED HEATING

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The temperature-relaxation functions of the stress, internal energy, and heat flux required in estimating the thermal stress in metals, in conditions of intense pulsed heating for arbitrary and small time intervals, are found.

Processes occurring in the fast action of intense heat fluxes on solid media are accompanied by thermal and thermomechanical relaxational phenomena, which must be taken into account in estimating the stress. In [1, 2], the solution was obtained for viscoelastic media with thermal memory, when relaxation is due solely to thermal perturbation. The given dependences of the stress and temperature fields on the function (core) of the relaxing parameters of the medium in the estimates may be used with known cores. The explicit form of the function is also required for the analysis of features of the solutions obtained in [2]. Methods of nonequilibrium thermodynamics were used in [3, 4] to obtain the constraints imposed on the functions used in the calculations, the explicit form of which may be established using physical and physiocomechanical models of the behavior of the given media.

Arbitrary Time Intervals

For metallic polycrystals, the model of a linear body of [5] with relaxation cores R exp $(-\tau + s/\tau_r)$ is applicable. Experiment indicates that the exponential includes a certain function depending on the form of reaction of the medium to an external perturbation varying in the interval [0, 1]. In the absence of relaxation, this factor is zero, and for the model [5] it is unity. It is usually determined empirically, and divergence of the factor from unity may be explained by the approximate nature of the model of the relaxation processes.

The relaxing terms of the system of equations in [1, 2] may be written in the form

$$\int_0^\infty \varphi(s) F(\tau - s)\, ds = R \int_0^\tau \exp\left(A \frac{\tau - s}{\tau_r}\right) F(\tau - s)\, ds, \tag{1}$$

where $s \leqslant \tau$; $0 \leq \tau \leqslant \tau_*$; $\tau_* \to \infty$; $F(\tau - s)$ is some function of the temperature, and $\varphi(\tau - s)$ is the relaxation core, which may be explicitly expressed when R and τ_r (the spectra of relaxing parameters and relaxation times, respectively) are known and is determined by the properties of the medium and the perturbation source, and therefore Eq. (1) must be written in the form

$$\int_0^\infty \varphi(s) F(\tau - s)\, ds = \int_0^\tau \sum_n R_n \exp\left(-A_{rn} \frac{\tau - s}{\tau_{rn}}\right) F(\tau - s)\, ds. \tag{2}$$

In [6], the heat flux in the medium is described as follows in a linear approximation

$$q_i = -\int_0^\tau \alpha(\tau - s)\, t_{,i}(\tau - s)\, ds \tag{3}$$

when $s \leqslant \tau$, $0 \leq \tau \leqslant \tau_*$, $\tau_* \to \infty$. In Eq. (3), $\alpha(\tau - s)$ is the relaxation core of the flow. Next Eq. (3) is written in the form

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$$q_i = -R_q \int_0^{\tau} \sum_n \exp\left(-\alpha_{qn} \frac{\tau - s}{\tau_{qn}}\right) t_{,i}(\tau - s)\, ds, \tag{4}$$

where $0 \leqslant \alpha_{qn} \leqslant 1$. In Eq. (4), R_q must be determined.

The next step is to establish the relation between the fluxes, stresses, and internal energies for nonrelaxing and relaxing media at times $\tau \geqslant 0$ (no relaxation) and $0 \leqslant \tau \leqslant \tau_{rn}$, $\tau > \tau_{rn}$ (relaxation observed). From all the τ_{rn}, the largest τ_r is chosen. The balance equation is written in the form

$$f = f^N - \Delta f = f^N - (f - f^R), \tag{5}$$

where f, f^R, f^N are the heat flux, stress, or internal energy at times $0 \leqslant \tau \leqslant \tau_{rnmax}$, $\tau > \tau_{rnmax}$ (with relaxation) and $\tau \geqslant 0$ when there is no relaxation.

According to Eq. (5), q_i is given by the following expressions at times $\tau \geqslant 0$ and $\tau > \tau_{qn}$

$$q_i^N = -\lambda^N t_{,i}, \quad q_i^R = -\lambda^R t_{,i}. \tag{6}$$

It follows from Eqs. (4)-(6) that

$$R_q = \frac{(\lambda^N + \lambda^R)\, t_{,i}}{2 \int_0^{\tau} \sum_n \exp\left(-\alpha_{qn} \frac{\tau - s}{\tau_{qn}}\right) t_{,i}(\tau - s)\, ds}, \tag{7}$$

and $\alpha(\tau - s)$ is determined from the relation

$$\alpha(\tau - s) = \frac{(\lambda^N + \lambda^R)\, t_{,i} \sum_n \exp\left(-\alpha_{qn} \frac{\tau - s}{\tau_{qn}}\right)}{2 \int_0^{\tau} \sum_n \exp\left(-\alpha_{qn} \frac{\tau - s}{\tau_{qn}}\right) t_{,i}(\tau - s)\, ds}, \tag{8}$$

where $\lambda^N = \lambda^R$ when $\tau = 0$.

In [1], the stress in the direction z is

$$\sigma_{zz} = (2\varkappa_3^M + \varkappa_4^M) u_z' + \varkappa_2^M t + \int_0^{\tau} \gamma(\tau - s)\, t(\tau - s)\, ds = (2\varkappa_3^M + \varkappa_4^M) u_z' - \varkappa_2^M t + R_\sigma \int_0^{\tau} \sum_n \exp\left(-\alpha_{\sigma n} \frac{\tau - s}{\tau_{\sigma n}}\right) t(\tau - s)\, ds \tag{9}$$

when $s \leqslant \tau$, $0 \leqslant \tau \leqslant \tau_*$, $\tau_* \to \infty$, where u_z' is the derivative of the displacement with respect to z. If stress relaxation as a result of shear is not present, Eqs. (5) and (9) give

$$\sigma_{zz}^N = (2\varkappa_3^M + \varkappa_4^M) u_z' - \varkappa_2^N t, \quad \sigma_{zz}^R = (2\varkappa_3^M + \varkappa_4^M) u_z' - \varkappa_2^R t, \tag{10}$$

where $\varkappa_2^N = \alpha_\theta E_0 (1 - \nu)^{-1}$. It follows from Eqs. (5), (9), and (10) that

$$R_\sigma = -\frac{(\varkappa_2^N + \varkappa_2^R - 2\varkappa_2^M)\, t}{\int_0^{\tau} \sum_n \exp\left(-\alpha_{\sigma n} \frac{\tau - s}{\tau_{\sigma n}}\right) t(\tau - s)\, ds}, \tag{11}$$

where $0 \leqslant \alpha_{\sigma n} \leqslant 1$, and the core $\gamma(\tau - s)$ is determined from the relation

$$\gamma(\tau - s) = -\frac{(\varkappa_2^N + \varkappa_2^R - 2\varkappa_2^M)\, t \sum_n \exp\left(-\alpha_{\sigma n} \frac{\tau - s}{\tau_{\sigma n}}\right)}{2 \int_0^{\tau} \sum_n \exp\left(-\alpha_{\sigma n} \frac{\tau - s}{\tau_{\sigma n}}\right) t(\tau - s)\, ds}. \tag{12}$$

In [6], the internal energy in the linear approximation is determined by the dependence

$$E = b + c_v^M t + \int_0^\tau \beta(\tau - s)\, t(\tau - s)\, ds \tag{13}$$

for $s \leqslant \tau$, $0 \leqslant \tau \leqslant \tau_*$, $\tau_* \to \infty$, where b is a constant. In Eq. (13), no account is taken of the component due to thermal expansion, which is given by the following expression in the elastic case, according to [7]

$$\alpha_{kl}^t c_{ijkl} l_{ij} t = \alpha_\theta E_0 (1 - 2\nu)^{-1} l t = \varkappa_2 l t, \tag{14}$$

where $l = \beta_\theta t$ is the dilatation. Taking account of Eq. (14), the internal energy is written in the form

$$E = b + c_v^M t + \varkappa_2^M \beta_\theta t^2 + R_e \int_0^\tau \sum_n \exp\left(-\alpha_{en} \frac{\tau - s}{\tau_{en}}\right) t(\tau - s)\, ds. \tag{15}$$

For the unrelaxed and relaxed internal energy, Eqs. (5) and (15) give

$$E^N = b + \overset{N}{c_v} t + \overset{N}{\varkappa_2} \beta_\theta t^2, \quad E^R = b + \overset{R}{c_v} t + \overset{R}{\varkappa_2} \beta_\theta t^2, \tag{16}$$

where relaxation in α_θ and β_θ is neglected. In Eq. (16), $c_v^N = c_{v0}$ is the specific heat at $\Theta = t_0$. From Eqs. (5), (15), and (16), the following expression is obtained for R_e

$$R_e = \frac{(c_v^N + c_v^R - 2c_v^M)\, t + (\varkappa_2^N + \varkappa_2^R - 2\varkappa_2^M)\, \beta_\theta t^2}{2 \int_0^\tau \sum_n \exp\left(-\alpha_{en} \frac{\tau - s}{\tau_{en}}\right) t(\tau - s)\, ds}, \tag{17}$$

and according to Eqs. (15) and (17)

$$\beta(\tau - s) = \frac{(c_v^N + c_v^R - 2c_v^M)\, t + (\varkappa_2^N + \varkappa_2^R - 2\varkappa_2^M)\beta_\theta t^2}{\int_0^\tau \sum_n \exp\left(-\alpha_{en} \frac{\tau - s}{\tau_{en}}\right) t(\tau - s)\, ds} \sum_n \exp\left(-\alpha_{en} \frac{\tau - s}{\tau_{en}}\right) \tag{18}$$

where $0 \leqslant \alpha_{en} \leqslant 1$.

Small Time Intervals

The relaxation function is found at times $\tau < \tau_{rnmin}/A_{rn}$, when only the first two terms need be retained in the series expansion of the exponential and τ_{rnmin} is the smallest τ_{rn}.

The core $\varphi(\tau - s)$ in Eq. (2) may be written in the form of a series

$$\varphi(\tau - s) = \varphi(\tau) + \varphi'(\tau) \sum_n A_{rn} \frac{\tau - s}{\tau_{rn}} + O\left[\left(A_{rn} \frac{\tau - s}{\tau_{rn}}\right)^2\right], \tag{19}$$

where $\varphi(\tau)$ and $\varphi'(\tau)$ are determined from Eq. (2) under the following conditions

$$\varphi(\tau) F(\tau) = \int_0^\tau \sum_n \psi_n(\tau) F(\tau - s)\, ds = \zeta^N F(\tau),$$
$$\varphi_n'(\tau) F(\tau) = \int_0^\tau \psi_n'(\tau) F(\tau - s)\, ds = \Delta\zeta_n F(\tau), \tag{20}$$

where ζ^N, $\Delta\zeta_n$ are, respectively, the total unrelaxed parameter and the n-th order in relaxation, while $\psi_n(\tau)$ and $\psi_n'(\tau)$ are the coefficients of the zero and first approximation in the expansion of the exponentials. From Eq. (20)

$$\varphi(\tau) = \zeta^N (N\tau)^{-1}, \quad \varphi'(\tau) = 2\Delta\zeta \tau^{-1}, \tag{21}$$

where N is the number of relaxing parameters. The following results are obtained from Eqs. (4), (9), (15), and (21) for the heat flux, stress, and internal energy

$$q_i = \int_0^\tau \left[\lambda^{\mathrm{N}} (N\tau)^{-1} - 2 \sum_n \Delta\lambda_n \alpha_{qn} \frac{\tau - s}{\tau\tau_{qn}} \right] t_{,i} (\tau - s)\, ds, \tag{22}$$

$$\sigma_{zz} = (2\varkappa_3^{\mathrm{M}} + \varkappa_4^{\mathrm{M}}) u_z' - \int_0^\tau [\varkappa_2^{\mathrm{M}} (N\tau)^{-1} - 2 \sum_n \Delta\varkappa_{2n} \alpha_{\sigma n} \frac{\tau - s}{\tau_{\sigma n}}] t (\tau - s)\, ds, \tag{23}$$

$$E = b + \int_0^\tau \left[\frac{c_v^{\mathrm{M}} + \varkappa_2^{\mathrm{M}} \beta_\theta t}{N\tau} - 2 \sum_n (\Delta c_{vn} + \Delta\varkappa_{2n} \beta_\theta t) \alpha_{en} \frac{\tau - s}{\tau_{en}} \right] t (\tau - s)\, ds. \tag{24}$$

Suppose the only grain-boundary relaxation of the heat flux and relaxation in the elastic modulus are perceptible. Then it follows from Eqs. (22)-(24) that

$$q_i = \int_0^\tau \left[\lambda^{\mathrm{N}} \tau^{-1} - \frac{2}{3} \left(\pi \frac{k}{e} \right)^2 \left(\frac{A\sigma}{K} \right)^2 \Theta\Omega^{-1} \frac{\tau - s}{\tau\tau_q} \right] t_{,i} (\tau - s)\, ds, \tag{25}$$

$$\sigma_{zz} = (2\varkappa_3^{\mathrm{M}} + \varkappa_4^{\mathrm{M}}) u_z^1 - \int_0^\tau \left(\varkappa_2^{\mathrm{M}} \tau^{-1} - \frac{E_0^2 \Theta \alpha_\theta^2}{c_v + E_0 \alpha_\theta^2 \Theta} \frac{\tau - s}{\tau\tau_\sigma} \right) t (\tau - s)\, ds, \tag{26}$$

$$E = b + \int_0^\tau \left[\frac{c_v}{\tau} + \frac{\alpha_\theta \beta_\theta E_0 \Theta}{(1 - 2\nu)\tau} - \frac{\alpha_\theta^3 \beta_\theta E_0^2 \Theta^2 (\tau - s)}{(c_v + E_0 \Theta \alpha_\theta^2) \tau\tau_e} \right] t (\tau - s)\, ds, \tag{27}$$

where the relaxation functions at small times are approximate expressions

$$\alpha (\tau - s) \simeq \lambda^{\mathrm{N}} \tau^{-1} - \frac{2}{3} \left(\pi \frac{k}{e} \right)^2 \left(\frac{A\sigma}{K} \right)^2 \Omega^{-1} \Theta \frac{\tau - s}{\tau\tau_q}, \tag{28}$$

$$\gamma (\tau - s) \simeq \varkappa_2^{\mathrm{M}} \tau^{-1} - \frac{E_0^2 \Theta \alpha_\theta^2}{c_v + E_0 \alpha_\theta^2 \Theta} \frac{\tau - s}{\tau\tau_\sigma}, \tag{29}$$

$$\beta (\tau - s) \simeq c_v \tau^{-1} + \frac{\alpha_\theta \beta_\theta E_0 \Theta}{(1 - 2\nu)\tau} - \frac{\alpha_\theta^3 \beta_\theta E_0^2 \Theta^2}{c_v + E_0 \Theta \alpha_\theta^2} \frac{\tau - s}{\tau\tau_e}. \tag{30}$$

In Eqs. (25)-(30), $\tau_q = \tau_\sigma = \tau_e = d^2/a$. In Eq. (25), the dimensional defect of the thermal conductivity

$$\Delta\lambda = \lambda^{\mathrm{N}} - \lambda^{\mathrm{R}} = \frac{\pi^2}{3} \left(\frac{k}{e} \right)^2 \left(\frac{A\sigma}{K} \right)^2 \Omega^{-1} \Theta$$

is a consequence of the Wiedemann–Franz law $\lambda = (\pi^2/3)(k/e)^2 \theta\Omega^{-1}$ and the dependence of the electrical conductivity of the polycrystal on the hydrostatic pressure according to [8]: $\Delta\Omega/\Omega \simeq (K/A\sigma)^2$.

In [5], the dimensionless defect of the elastic modulus Δ_E, associated with temperature relaxation, was given in the form

$$\Delta_E = \frac{E^{\mathrm{N}} - E^{\mathrm{R}}}{E^{\mathrm{R}}} = E^{\mathrm{N}} \theta \frac{\alpha_\theta^2}{c_v},$$

where $E^{\mathrm{R}} = \varkappa^{\mathrm{R}}_2$ and $E^{\mathrm{N}} = E_0$, and hence the dimensional defect of the elastic modulus used in Eqs. (26) and (27) is

$$\Delta E = E^{\mathrm{N}} - E^{\mathrm{R}} = \Delta\varkappa_2 = E_0 \Theta \frac{\alpha_\theta^2}{c_v \left(1 + E_0 \Theta \frac{\alpha_\theta}{c_v} \right)}.$$

In Eqs. (25)-(30), according to the conditions, N = 1 and it is assumed that $\alpha_{qn} = \alpha_{\sigma n} = \alpha_{en} = 1$. The relations obtained for the heat flux, stress, internal energy, and relaxing cores allow the thermal stress arising in metals under thermal impact to be estimated if the properties of the relaxing parameters are known. Since the stress plays a significant role only in a very thin surface layer, Eqs. (28)-(30) may be used in the estimates, satisfying the conditions of small time intervals.

NOTATION

θ, t_0, current and equilibrium temperature, °K; $t_{,i} = (\theta - t_0)_{,i}$, temperature gradient in the direction i; q_i, heat flux density; E, internal energy density; σ_{zz}, normal stress in the direction z; λ, thermal conductivity; c_v, specific heat; a, thermal diffusivity; Ω, electrical resistivity; K, modulus of omnidirectional compression; E_0, Young's modulus; α_θ, β_θ, coefficients of linear and volume thermal expansion; ν, Poisson's ratio; σ, characteristic grain-boundary stress; e, electron charge; k, Boltzmann constant; A, constant of the medium; d, linear grain dimension; u_z, displacement in the direction z; γ, α, β, temperature-relaxation cores of the stress, heat flux, and internal energy; f^N, f^R, nonrelaxed and relaxed characteristics of the medium; f^M, momentary value of the characteristics of the medium; τ, s, time; τ_{qn}, τ_{en}, $\tau_{\sigma n}$, relaxation times of heat flux, internal energy, and stress determined by the n parameter; $\varkappa_i$ (i = 2, 3, 4), physicomechanical characteristics of the medium. Indices: N, nonrelaxed; R, relaxed.

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